Frequency Domain Properties of EALQR with Indefinite Weighting Matrix

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EALQR (Linear Quadratic Regulator design with Eigenstructure Assignment capability) has the capability of exact assignment of eigenstructure with the guaranteed margins of the LQR for MIMO (Multi-Input Multi-Output) systems. However, EALQR undergoes a restriction on the state-weighting matrix Q in LQR to be indefinite with respect to the region of allowable closed-loop eigenvalues. The definiteness of the weighting matrix is closely related to the robustness property of a given system. In this paper, we derive a relation between the indefinite weighting matrix Q and the robustness property for EALQR. The modified frequency domain inequality, that could be guaranteed by EQLQR with an indefinite weighting matrix, is presented.

Key Words: Eigenstructure Assignment, LQR, Frequency Domain Properties, Flight Control System

1. Introduction

To date many researchers have developed eigenvalue assignment methods which make use of LQR (Broussard, 1982; Harvey and Stein, 1978; Hiroe et al., 1973; Innocenti and Stanziola, 1990; Ochi and Kanai, 1993; Wilson and Cloutier, 1990). However, they did not directly consider the problem of eigenstructure assignment (Choi et al., 1992; Choi, 1998; Siouris et al., 1995) in their papers. Recently, Choi and Seo (1999) introduced the EALQR (Linear Quadratic Regulator design with Eigenstructure Assignment capability) that has the capability of exact assignment of eigenstructure with the guaranteed margins of LQR (Dorato et al., 1995; Siouris, 1996) for MIMO (Multi-Input Multi-Output) systems. EALQR is based on a transformation method using a block controller in order to cope with the rank deficiency problem in the control input matrix. One feature of the proposed method is that the state-weighting matrix Q is obtained by solving simple matrix equations which are derived from the matrix Riccati equation of the Hamilton matrix of LQR. Another feature of the method is that it can place eigenvalues arbitrarily and exactly at the desired locations as well as eigenvectors in the least square sense according to the conditions of the given system.

However, EALQR undergoes a restriction on the state-weighing matrix Q in LQR to be indefinite with respect to the region of allowable closed-loop eigenvalues. In some literatures (Al-Sumi and Stevens, 1992; Fujii, 1987; Kenji, 1998; Luo and Lan, 1995; Molinari, 1981; Ohta et al., 1991), the effects of the indefinite Q in LQ problem are analyzed. Though they have shown that Q does not have to satisfy the definiteness condition for the existence of a solution (Al-Sumi and Stevens, 1992; Molinari, 1981; Ohta et al., 1991), the definiteness of the weighting matrix Q is closely related to the robustness property of a given system. In those cases, the frequency domain inequality in MIMO systems, which is the circle condition (Choi and Seo, 1999) in SISO

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(Single-Input Single-Output) system, using the Nyquist stability theorem (Dorato et al., 1995) does not completely hold. However EALQR does not guarantee the definiteness of the weighting matrix, it is required to investigate further study on the robustness of the EALQR with the indefinite weighting matrix.

In this paper, we derive a relation between the indefinite state-weighting matrix Q and the robustness property for EALQR. The modified frequency domain inequality, that is guaranteed by EQLQR with an indefinite weighting matrix, using the Spectral theorem for Hermitian matrices (Horn and Johnson, 1985) is presented. The frequency domain stability-robustness bound is calculated for the case of a flight control system design example.

2. Problem Formulation

Consider a linear time invariant multi-variable controllable system

$$\dot{x}=Ax+Bu \tag{1}$$

where x, u denote the n, m dimensional state variable and control input vectors, respectively. A and B are system and input matrices with appropriate dimensions, respectively. In EALQR, the given system should be transformed to a block controllable form in order to cope with the rank deficiency problem in the control input matrix as follows:

$$\bar{A}=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ \bar{B}=\begin{bmatrix} I_m \\ 0 \end{bmatrix} \tag{2}$$

where A_{11}, A_{12}, A_{21}, A_{22}, and I_m are $m\times m$, $(n-m)\times m$, $m\times(n-m)$, $(n-m)\times(n-m)$ dimensional submatrices of the given system, and $m\times m$ dimensional identity matrix, respectively.

The objective of the EALQR problem is to find an optimal control input u^* that minimizes the following cost function and satisfies the required conditions for locating the desired eigenvalues and eigenvectors.

$$\frac{1}{2}\int_0^\infty (x^TQx + u^TRu)\,dt \tag{3}$$

$$(\bar{A}-\bar{B}K)\phi_i=\lambda_i\phi_i,\ i=1, 2, \cdots, n \tag{4}$$

where Q, R, λ_i, and ϕ_i are a state weighting matrix, an input weighting matrix, an ith closed-loop eigenvalue, and the corresponding closed-loop eigenvector, respectively. The gain matrix, $K=R^{-1}\bar{B}^TP=[K_1\ \ K_2]$, of EALQR can be obtained by solving the following block matrix Riccati equation.

$$P\bar{A}+\bar{A}^TP-P\bar{B}R^{-1}\bar{B}^TP+Q=0 \tag{5}$$

where P has the following form:

$$P=\begin{bmatrix} RK_1 & RK_2 \\ RK_2^T & I_m \end{bmatrix} \tag{6}$$

where K_1 and K_2 are the submatrices that contain information on the desired eigenstructure.

The Kalman equation, derived from the block matrix Riccati equation, is given by

$$\begin{aligned}&[I+K\Phi(-s)\bar{B}]^TR[I+K\Phi(s)\bar{B}]\\&=R+\bar{B}\Phi^T(-s)Q\Phi(s)\bar{B}\end{aligned} \tag{7}$$

where $\Phi(s)=(sI-\bar{A})^{-1}$. If the matrix Q is a positive semi-definite, the circle condition with $s=j\omega$ is satisfied for all ω such that

$$[I+K\Phi(-j\omega)\bar{B}]^TR[I+K\Phi(j\omega)\bar{B}]\geq R \tag{8}$$

But, the circle condition is changed in case where the weighting matrix has some negative eigenvalues.

The EALQR controller has low-sensitivity for all frequency ranges because the matrix $[I+K\Phi(j\omega)\bar{B}]$ is the inverse of the sensitivity matrix. In particular, the gain margin of the loop transfer function $K\Phi(s)\bar{B}$ of a single-input system is infinite, and the phase margin is greater than 60 degrees.

In EALQR, the achieved Q from the block matrix Riccati equation is not always guaranteed to be positive semi-definite. If Q is indefinite, all the eigenvalues of $\bar{B}^T\Phi^T(-s)Q\Phi(s)\bar{B}$ cannot be guaranteed to be positive or zero. If EALQR does not have a guaranteed circle condition, the merit of the EALQR control methodology will shrink, although it may be possible to assign a desired eigenstructure arbitrarily. Thus, we derive a relation between the indefinite state-weghting matrx Q and the robustness property for EALQR.

3. The Modified Frequency Domain Inequality

3.1 Background

The matrix singular value is defined as

$$\sigma(A)=\sqrt{\lambda(A^{*}A)} \tag{9}$$

where $\sigma(\cdot)$ is a singular value of the matrix $(\cdot)$; $(\cdot)^{*}$ denotes the complex conjugate transpose of $(\cdot)$; and $\lambda(A^{*}A)$ are eigenvalues of the matrix $A^{*}A$.

To decompose an indefinite Q into two submatrices, *i.e.*, the submatrix that contains only positive eigenvalues and the submatrix that contains only negative eigenvalues, we decompose the matrix Q by using the spectral decomposition as follows:

$$Q=\Phi\Lambda\Psi^{T} \tag{10}$$

where Φ, Ψ, and Λ are a right modal matrix, a left modal matrix, and a diagonalized matrix with eigenvalues of Q in its diagonal, respectively.

Theorem 1 (Spectral theorem for Hermitian matrices) (Horn and Johnson, 1985)

Let $Q\in M_n$ be given, and M_n is a set of $n\times n$ dimensional matrices. Then, $Q=\Phi\Lambda\Psi^{T}=\Phi\Lambda\Phi^{T}$ if and only if the matrix Q is real and Hermitian, where $\Psi\in M_n$ and $\Lambda\in M_n$ are a unitary matrix and a real diagonal matrix, respectively.

Based on the above theorem, we can get the following theorem.

Theorem 2

Let $Q\in M_n$ be given. Then,

$$\lambda\Big[\sum_{i=1}^{n}(\phi_i\lambda_i\phi_i^{T})\Big]=\sum_{i=1}^{n}\big[\lambda(\phi_i\lambda_i\phi_i^{T})\big]$$

Proof :

Let us decompose the Q using spectral decomposition as follows:

$$Q\equiv\Phi\Lambda\Phi^{T}=\sum_{i=1}^{n}\phi_i\lambda_i\phi_i^{T}$$

Then,

$$\begin{aligned}\lambda(\Phi\Lambda\Phi^{T})&=\lambda[\Phi\sqrt{\Lambda}^{T}\sqrt{\Lambda}\Phi^{T}]\\&=\lambda[(\sqrt{\Lambda}\Phi^{T})^{T}(\sqrt{\Lambda}\Phi^{T})]\\&=\lambda\Big[\sum_{i=1}^{n}(\sqrt{\lambda_i}\phi_i^{T})^{T}(\sqrt{\lambda_i}\phi_i^{T})\Big]\\&=\sigma^{2}\Big[\sum_{i=1}^{n}\sqrt{\lambda_i}\phi_i^{T}\Big]\\&=\sum_{i=1}^{n}\sigma^{2}[\sqrt{\lambda_i}\phi_i^{T}]\\&=\sum_{i=1}^{n}\lambda[(\sqrt{\lambda_i}\phi_i^{T})^{T}(\sqrt{\lambda_i}\phi_i^{T})]\end{aligned}$$

Thus,

$$\lambda\Big[\sum_{i=1}^{n}(\phi_i\lambda_i\phi_i^{T})\Big]=\sum_{i=1}^{n}\big[\lambda(\phi_i\lambda_i\phi_i^{T})\big]$$

3.2 The frequency domain inequality for LQR

Based on the multivariable *Nyquist stability theorem* (Dorato et al., 1995), the frequency domain equality for LQR is derived as

$$[I+G(-s)]^{T}R[I+G(s)]=R+H(s) \tag{11}$$

where $G(s)=R^{-1}\bar{B}^{T}P(sI-\bar{A})^{-1}\bar{B}$, and $H(s)=[(-sI-\bar{A})^{-1}\bar{B}]^{T}Q[(sI-\bar{A})^{-1}\bar{B}]$.

If $Q\geq 0$, then all the eigenvalues of $H(s)$ are greater than 0, *i.e.*, all the singular values of $H(s)$ are always greater than 0. Thus, the frequency domain inequality is given as

$$[I+G(-s)]^{T}R[I+G(s)]\geq R \tag{12}$$

If we consider a design method with $R=\rho I$, $\rho>0$ in Eq. (12), then we obtain

$$[I+G(-s)]^{T}[I+G(s)]\geq 1 \tag{13}$$

Finally, the (Kalman) inequality can be derived as

$$\sigma_{\min}[I+G(s)]\geq 1 \text{ (0 dB)} \tag{14}$$

Eq. (14) satisfies the frequency domain properties of the standard LQR.

Now, we decompose $H(s)$ into the following form by using Theorem 1.

$$\begin{aligned}H(s)&=\bar{B}^{T}(-sI-\bar{A})^{-T}\Phi\Lambda\Phi^{T}(sI-\bar{A})^{-1}\bar{B}\\&=\sum_{i=1}^{n}\bar{B}^{T}(-sI-\bar{A})^{-T}\phi_i\lambda_i\phi_i^{T}(sI-\bar{A})^{-1}\bar{B}\end{aligned} \tag{15}$$

The eigenvalues of $H(s)$ are equal to the sum of the eigenvalues with each decomposed term by using Theorem 2, *i.e.*,

$$\lambda[H(s)]=\lambda[\bar{B}^T(-sI-\bar{A})^{-T}\Phi\Lambda\Phi^T(sI-\bar{A})^{-1}\bar{B}] \\ =\sum_{i=1}^{n}\lambda[\bar{B}^T(-sI-\bar{A})^{-T}\phi_i\lambda_i\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \quad (16)$$

where $\lambda[H(s)]$ are eigenvalues of the matrix $H(s)$.

If all the eigenvalues of Λ are positive, then $\Lambda=\sqrt{\Lambda}^T\sqrt{\Lambda}$. Thus, we can rewrite Eq. (16) as follows:

$$\sigma^2[\sqrt{\Lambda}\Phi^T(sI-\bar{A})^{-1}\bar{B}] \\ =\sum_{i=1}^{n}\sigma_i^2[\sqrt{\lambda_i}\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \quad (17)$$

Since EALQR with a positive semi-definite Q in the performance index always yields Eq. (17), both $\sigma^2[\sqrt{\Lambda}\Phi^T(sI-\bar{A})^{-1}\bar{B}]\geq 0$ and $H(s)>0$ are also guaranteed. But, if Q has a negative eigenvalue, *i.e.* Q is indefinite, then $H(s)\geq 0$ is not guaranteed.

In this case, indefinite Q may not yield Eq. (17) because the indefinite matrix cannot be decomposed in terms of square.

3.3 The modified frequency domain inequality for EALQR

Given an $n\times n$ indefinite matrix Q, it can be represented using Theorem 1 as:

$$Q=\phi_1\lambda_1\phi_1^T+\phi_2\lambda_2\phi_2^T+\cdots+\phi_n\lambda_n\phi_n^T \quad (18)$$

Let the $r(\leq n)$ eigenvalues be positive, then Eq. (18) can be rewritten as

$$Q=(\phi_1\lambda_1\phi_1^T+\cdots+\phi_r\lambda_r\phi_r^T) \\ +(\phi_{r+1}\lambda_{r+1}\phi_{r+1}^T+\cdots+\phi_n\lambda_n\phi_n^T) \quad (19)$$

where the $n-r$ eigenvalues from $r+1$ to n are have negative values. Thus, Eq. (19) can be expressed as

$$Q=(\phi_1\lambda_1\phi_1^T+\cdots+\phi_r\lambda_r\phi_r^T) \\ -(-\phi_{r+1}\lambda_{r+1}\phi_{r+1}^T-\cdots-\phi_n\lambda_n\phi_n^T) \quad (20)$$

From Eq. (20),

$$Q=\sum_{i=1}^{r}\phi_i\lambda_i\phi_i^T-\sum_{i=r+1}^{n}\phi_i(-\lambda_i)\phi_i^T \quad (21)$$

where $\lambda_i>0$, $i=1,\cdots,r$ and $-\lambda_i>0$, $i=r+1,\cdots,n$. Therefore, in case of EALQR with indefinite Q, $\lambda[H(s)]$ is obtained as follows:

$$\lambda[H(s)]=\sum_{i=1}^{n}\lambda[\bar{B}^T(sI-\bar{A})^{-T}\Phi\Lambda\Phi^T(sI-\bar{A})^{-1}\bar{B}] \\ =\sum_{i=1}^{r}\lambda[\bar{B}^T(-sI-\bar{A})^{-T}\phi_i\lambda_i\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \\ -\sum_{i=r+1}^{n}\lambda[\bar{B}^T(-sI-\bar{A})^{-T}\phi_i(-\lambda_i)\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \quad (22)$$

Moreover, Eq. (22) can be expressed using a Theorem 2 as:

$$\lambda[H(s)]=\sum_{i=1}^{r}\sigma^2[\sqrt{\lambda_i}\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \\ -\sum_{i=r+1}^{n}\sigma^2[\sqrt{-\lambda_i}\phi_i^T(sI-\bar{A})^{-1}\bar{B}] \quad (23)$$

In case of $\lambda[H(s)]>0$, from Eq. (9) and Eq. (11), we have

$$\sigma_{\min}(\sqrt{R}[I+G(s)])\geq\sigma_{\max}(\sqrt{R+H(s)}) \quad (24)$$

where, the input weighting matrix R is supposed to be the approximate identity matrix. Eq. (24) becomes as follows:

$$\sigma_{\min}(\sqrt{R}[I+G(s)])\approx\sigma_{\min}(\sqrt{R})\sigma_{\min}[I+G(s)] \quad (25)$$

Moreover, from the property of singular value decomposition (Horn and Johnson, 1985)

$$\sigma_{\max}[\sqrt{R+H(s)}]=\sqrt{\sigma_{\max}^2(\sqrt{R})+\lambda[H(s)]} \quad (26)$$

Eq. (24) can be rewritten using Eqs. (25) and (26) as

$$\sigma_{\min}(\sqrt{R})\sigma_{\min}[I+G(s)]\geq\sqrt{\sigma_{\max}^2(\sqrt{R})+\lambda[H(s)]} \quad (27)$$

From Eq. (27), we get

$$\sigma_{\min}[I+G(s)]\geq\sqrt{\frac{\sigma_{\max}^2(\sqrt{R})}{\sigma_{\min}^2(\sqrt{R})}+\frac{\lambda[H(s)]}{\sigma_{\min}^2(\sqrt{R})}} \quad (28)$$

Finally, we can conclude that frequency domain inequality of EALQR with indefinite Q is derived if and only if $\frac{\sigma_{\max}^2(\sqrt{R})}{\sigma_{\min}^2(\sqrt{R})}+\frac{\lambda[H(s)]}{\sigma_{\min}^2(\sqrt{R})}$ in Eq. (28) is positive.

3.4 The modeling error bound

Since the sensitivity TFM (Transfer Function Matrix), $S(s)$, is $[I+G(s)]^{-1}$, we can obtain the following inequality:

$$\sigma_{\max}[S(j\omega)]\leq\left(\sqrt{\frac{\sigma_{\max}^2(\sqrt{R})}{\sigma_{\min}^2(\sqrt{R})}+\frac{\lambda[H(j\omega)]}{\sigma_{\min}^2(\sqrt{R})}}\right) \quad (29)$$

Eq. (29) means that the maximum singular value of EALQR sensitivity TFM is always smaller than $20\log_{10}\left(1/\sqrt{\frac{\sigma^2_{max}(\sqrt{R})}{\sigma^2_{min}(\sqrt{R})}+\frac{\lambda[H(j\omega)]}{\sigma^2_{min}(\sqrt{R})}}\right)$ [dB].

Using the relationship for $G(s)$, the closed-loop TFM $(C(s)=(1+T(s))^{-1}T(s))$ is given by

$$\sigma_{max}[C(j\omega)]\leq 1+\left(1/\sqrt{\frac{\sigma^2_{max}(\sqrt{R})}{\sigma^2_{min}(\sqrt{R})}+\frac{\lambda[H(j\omega)]}{\sigma^2_{min}(\sqrt{R})}}\right) \quad (30)$$

Eq. (30) means that the robustness of EALQR is guaranteed under the following modeling error bound,

$$\sigma_{max}[E(j\omega)]<\frac{\sqrt{\frac{\sigma^2_{max}(\sqrt{R})}{\sigma^2_{min}(\sqrt{R})}+\frac{\lambda[H(j\omega)]}{\sigma^2_{min}(\sqrt{R})}}}{1+\sqrt{\frac{\sigma^2_{max}(\sqrt{R})}{\sigma^2_{min}(\sqrt{R})}+\frac{\lambda[H(j\omega)]}{\sigma^2_{min}(\sqrt{R})}}}$$

where $E(j\omega)=G_N(j\omega)^{-1}[G_A(j\omega)-G_N(j\omega)]$. $G_A(j\omega)$ and $G_N(j\omega)$ denote actual and nominal system, respectively.

4. Application to a Flight Control System Design

A linear model of a fighter aircraft under consideration is the following linearized two-input 4th-order continuous controllable system (Choi and Seo, 1999; Luo and Lan, 1995). The aircraft is trimmed at $Mach=1.5$ and $h=10{,}000$ *ft*. The assumed angle of attack is $\alpha=0.86$ deg. The locally linearized lateral directional equations of motion are given by

$$\begin{bmatrix}\dot\beta\\ \dot p\\ \dot r\\ \dot\phi\end{bmatrix}=\begin{bmatrix}-0.493 & 0.015 & -1 & 0.02\\ -61.176 & -7.835 & 4.991 & 0\\ 31.804 & -0.235 & -0.994 & 0\\ 0 & 1 & -0.015 & 0\end{bmatrix}\begin{bmatrix}\beta\\ p\\ r\\ \phi\end{bmatrix}+\begin{bmatrix}-0.002 & 0.002\\ 8.246 & 1.849\\ 0.249 & -0.436\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_{df}\\ \delta_r\end{bmatrix}$$

where β, p, r, and ϕ represent the sideslip angle, roll rate, yaw rate, and roll angle, respectively. The δ_{df} and δ_r are the deflection angle of the differential flap and the rudder, respectively. The eigenvalues of the open-loop system are given by

$$\Lambda^{open}=[-0.7555\pm5.8067i,\ -7.8181,\ 0.007]$$

Let the desired closed-loop eigenvalues be -8.00 (roll), -0.05 (spiral), and $-4.88\pm3.66i$ (dutch roll). That is,

$$\Lambda^d=[-8,\ -4.88\pm3.66i,\ -0.05]$$

The desired left-modal matrix Ψ^d and its normalized form Ψ^d_{nor} are selected arbitrarily through the relationship of $\psi_i^T\psi_i=\delta_{ij}$. A guideline for determining the desired left-modal matrix is well described in Choi(1998). The specified Ψ^d and Ψ^d_{nor} are given by

$$\Psi^d=\begin{bmatrix}0.6 & 0.6+0.6i & 0.6-0.6i & 0.8737\\ 0.4 & 0.4+0.4i & 0.4-0.4i & 0.1263\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Psi^d_{nor}=\begin{bmatrix}0.8321 & 0.5883+0.5883i & 0.5883-0.5883i & 0.9897\\ 0.5547 & 0.3922+0.3922i & 0.3922-0.3922i & 0.1431\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

According to the design procedure of the EALQR algorithm (Choi and Seo, 1999), the achievable normalized left-modal matrix Ψ^a_{nor} is obtained in the least-square sense as follows:

$$\Psi^a_{nor}=\begin{bmatrix}-0.5473 & 0.1086+0.0454i & 0.1086-0.0454i & 0.9897\\ -0.082 & 0.1029-0.0032i & 0.1029+0.0032i & 0.1431\\ 0.654 & 0.6936-0.7005i & 0.6936+0.7005i & 0\\ -0.5158 & 0.0454-0.04i & 0.0454+0.04i & 0\end{bmatrix}$$

The direction of each vector of the resulting achievable left modal matrix is placed near the best possible direction of each desired left eigenvector in the least square sense, and the desired closed-loop eigenvalues are assigned exactly. The feedback gain matrix K and the weighting matrices Q and R are obtained, respectively, as follows:

$$K=\begin{bmatrix}-1.5459 & -0.0592 & 4.1769 & -0.6326\\ -2.9178 & 0.2789 & -17.0252 & 4.3767\end{bmatrix},$$

$$R=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \text{and}\ Q=\begin{bmatrix}-363.2 & 17.2 & -1145. & 292.7\\ 17.2 & -0.4 & 7 & -0.3\\ -1145. & 7 & 396.1 & -87.4\\ 292.7 & -0.3 & -87.4 & 19.6\end{bmatrix}$$

Eigenvalues of the weighting matrix Q are given by

$$\lambda(Q)=[-1218.7,\ 1270.2,\ 1.9,\ -1.4]$$

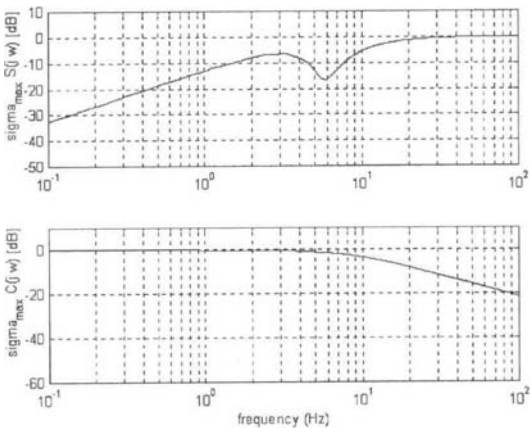

Fig. 1 Frequency domain properties of the designed flight control system

The maximum singular values of the sensitivity TFM ($S(j\omega)$) and the closed-loop TFM ($C(j\omega)$) in this case have the following bounds.

$$\begin{aligned}\sigma_{\max}[S(j\omega)] &= \frac{1}{\sqrt{1-0.0434}} \\ &= 1.0224 \\ &= 0.1927 \text{ [dB]}\end{aligned}$$

$$\begin{aligned}\sigma_{\max}[C(j\omega)] &= 2.0224 \\ &= 6.1173 \text{ [dB]}\end{aligned}$$

Thus, the resulting stability-robustness bound for the modeling error is given by

$$\sigma_{\max}[E(j\omega)] < 0.4945$$

Fig. 1 shows the frequency domain properties of the designed flight control system. In this case, Eq. (23) becomes negative. But, $\frac{\sigma^2_{\max}(\sqrt{R})}{\sigma^2_{\min}(\sqrt{R})} + \frac{\lambda[H(s)]}{\sigma^2_{\min}(\sqrt{R})}$ in Eq. (28) is positive, the changed frequency domain properties are negligible because each maximum singular value varies under each calculated bound.

5. Conclusions

The EALQR control design methodology has better performance than that of a conventional LQR or an eigenstructure assignment approach. It combines the respective advantages of both the LQR and the eigenstructure assignment while removing their crucial disadvantages. But, it has a constraint for the weighting matrix, that is, the weighting matrix in EALQR could be indefinite for some special systems. The definiteness of the weighting matrix is closely related to the robustness property of a given system.

In this paper, the effects of the indefinite Q in EALQR on the frequency domain properties are analyzed. The robustness criterion and quantitative frequency domain properties are also discussed. Finally, the frequency domain properties of EALQR have been analyzed by applying them to a flight control system design example.

Since there may exist cases where an achievable subspace of an eigenstructure does not belong to an achievable subspace of LQR, further discussions on the achievable subspace of both eigenstructure assignment and LQR should be exploited.

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